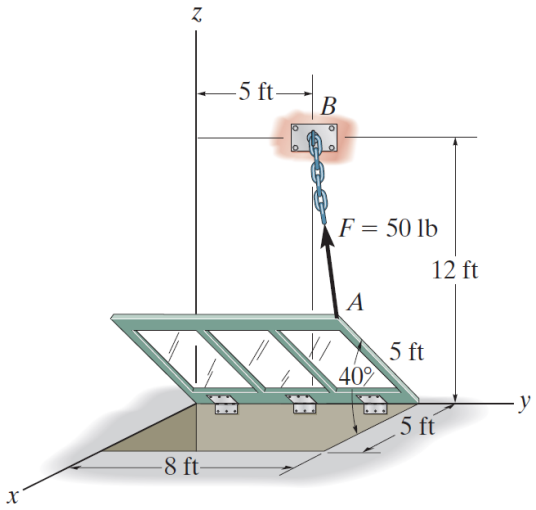
Problem 2-110

The window is held open by chain AB. Determine the length of the chain, and express the 50-lb force acting at A along the chain as a Cartesian vector and determine its coordinate direction angles.



Prob. 2–110

Solution

Write the position vectors to the points A and B.

$$\mathbf{r}_A = \langle 5\cos 40°, 8, 5\sin 40° \rangle \text{ ft}$$

$$\mathbf{r}_B = \langle 0, 5, 12 \rangle \text{ ft}$$

The position vector from A to B is then

$$\begin{aligned}\mathbf{r}_{AB} &= \mathbf{r}_B - \mathbf{r}_A \\ &= \langle -5\cos 40°, -3, 12 - 5\sin 40° \rangle \text{ ft}.\end{aligned}$$

Its magnitude is the length of the chain.

$$\begin{aligned}|\mathbf{r}_{AB}| &= \sqrt{(-5\cos 40°)^2 + (-3)^2 + (12 - 5\sin 40°)^2} \text{ ft} \\ &\approx 10.0 \text{ ft}\end{aligned}$$

Divide $\mathbf{r}_{AB}$ by its magnitude to get a unit vector in the same direction.

$$\hat{\mathbf{u}}_{AB} = \frac{\mathbf{r}_{AB}}{|\mathbf{r}_{AB}|} \approx \frac{\langle -5\cos 40°, -3, 12 - 5\sin 40° \rangle}{10.0} \quad \rightarrow \quad \begin{cases} \cos\alpha \approx \dfrac{-5\cos 40°}{10.0} \\ \cos\beta \approx \dfrac{-3}{10.0} \\ \cos\gamma \approx \dfrac{12 - 5\sin 40°}{10.0} \end{cases}$$

The direction angles for the force are therefore

$$\begin{cases} \alpha \approx 112^\circ \\ \beta \approx 107^\circ \\ \gamma \approx 29.0^\circ \end{cases}.$$

Finally, write the force $\mathbf{F}$.

$$\mathbf{F} = F\hat{\mathbf{u}}_{AB} \approx 50\,\frac{\langle -5\cos 40^\circ, -3, 12 - 5\sin 40^\circ \rangle}{10.0}\ \text{lb} \approx \langle -19.1, -14.9, 43.7 \rangle\ \text{lb}$$